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Rich Problem Solving Tasks

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Disciplines

Curriculum and Instruction | Science and Mathematics Education

Comments

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Rich Problem Solving Tasks

ICTM ISTS Conference 2014

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CCSSM Practice Standards

1. **Make sense of problems and persevere in solving**
2. **Reason abstractly and quantitatively.**
3. **Construct viable arguments and critique the reasoning of others.**
4. **Model with mathematics.**
5. **Use appropriate tools strategically.**
6. **Attend to precision.**
7. **Look for and make use of structure.**
8. **Look for and express regularity in repeated reasoning.**

Engaging Problems

Mathematics is like a video game;
If you just sit and watch,
you're wasting your quarter
(and semester).

Steve Benson

www.partycheap.com

* In under 2 minutes, can you think of any 4 odd numbers (including repeated numbers) that add up to 19?
* (Ball 2005, p. 26)

ODD + ODD = EVEN

EVEN + EVEN = EVEN

ODD + EVEN = ODD

EVEN + ODD = ODD

Engaging Problems

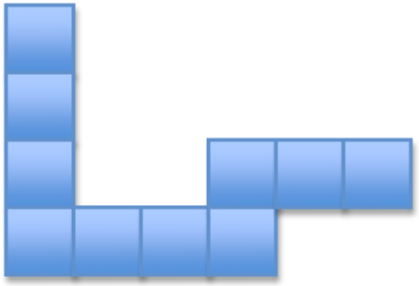
Tell me and I'll forget;

Show me and I may not remember;

Involve me and I'll understand.

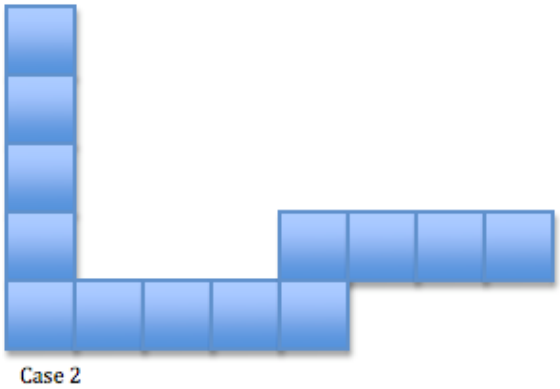
Native American Quote

Pattern Blocks

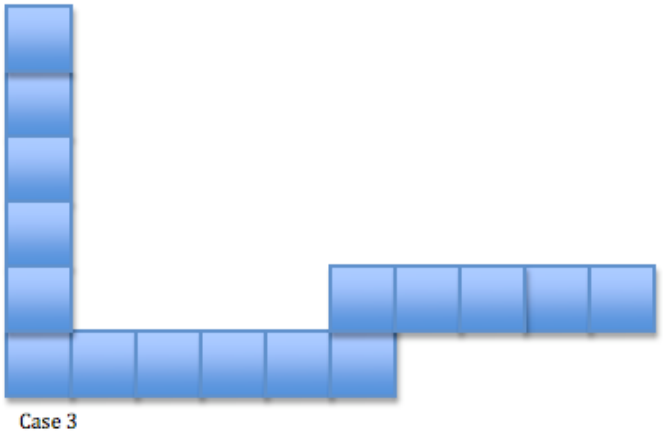


Case 1

Given the three block patterns, identify how the pattern is growing from Case 1 to Case 3.



Case 2



Case 3

Represent this growth

Stein, M. K., Smith, M. S., Henningsen, M. A., & Silver, E. A. (2000).

Pattern Blocks

* How did you approach this?

Engage Students

* Put at pairs on whiteboards
  * Don't have enough whiteboards, use the windows
* Hand-held whiteboards
* Foster discourse

Classroom Environment

* “establish classroom contexts in which questioning and proving are the norm” (Hodgson & Riley, 2001, p. 728).
* Adopt a “philosophy of getting them to ‘work it out for themselves’ in response to questions” (Benson & Findell, 2012, p. 2).

Ryan's Card Piles

* Let face cards = 10 and Aces = 1
* Deal 25 cards face up off the top of the deck
* Note the “lucky 7th card"
* Place the 25 cards at the bottom of the deck
* Deal 3 cards face up
* Add the value of the three cards, let it be *X*
* For each of the 3 cards, deal face down as many cards as it takes to get to 10
* Put all 3 piles at the bottom of the deck
* Deal *X* cards off the top of the deck, face up.
* The *X* + 1 card, will be the “lucky 7^{th} card"

Why does this work?

* Deal 25 cards face up off the top of the deck
* Note the “lucky 7th card" 7
* Place the 25 cards at the bottom of the deck 34
* Deal 3 cards face up 31

Why does this work?

* Deal 3 cards face up **31**
* Add the value of the three cards, let it be *X*
* For each of the 3 cards, deal face down as many cards as it takes to get to 10
* Put all 3 piles at the bottom of the deck

$$31 - (10-a) - (10-b) - (10-c)$$

$$31-30-(a+b+c)$$

$$1-X$$

Why does this work?

1-X

* Deal X cards off the top of the deck, face up. 1
* The X + 1 card, will be the "lucky 7th card"

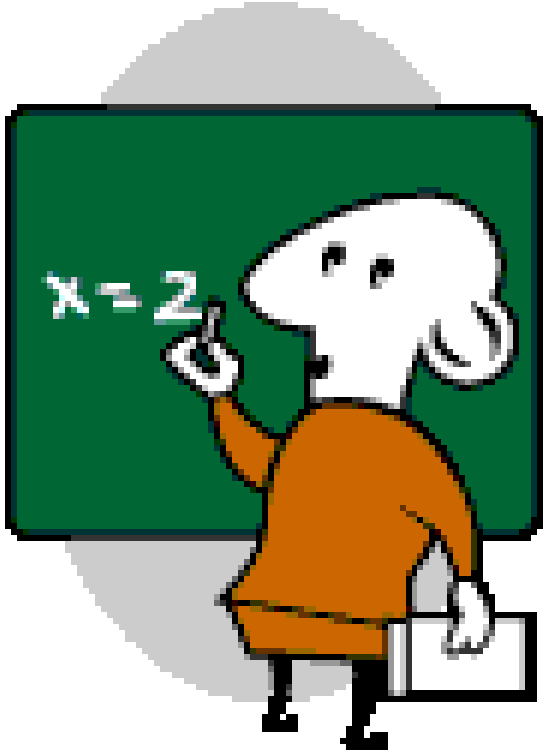
* Ann Halteman Focus on Math Seminar, December 3, 2008

Ryan's Card Piles

* What does this introduce?
  * Patterns
  * Algebra

Teacher-Centered

* The sage on the stage



Student-Centered

* Guide on the Side, not the sage on the stage
* A Danger of Social Constructivism

Content-Centered

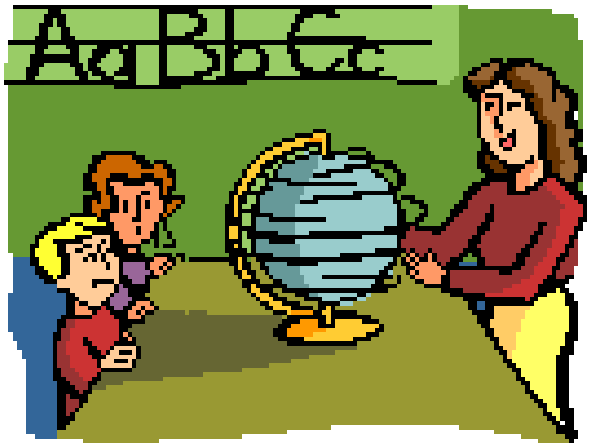
* Uses many of the same pedagogies as student-centered

What is the center of your classroom?

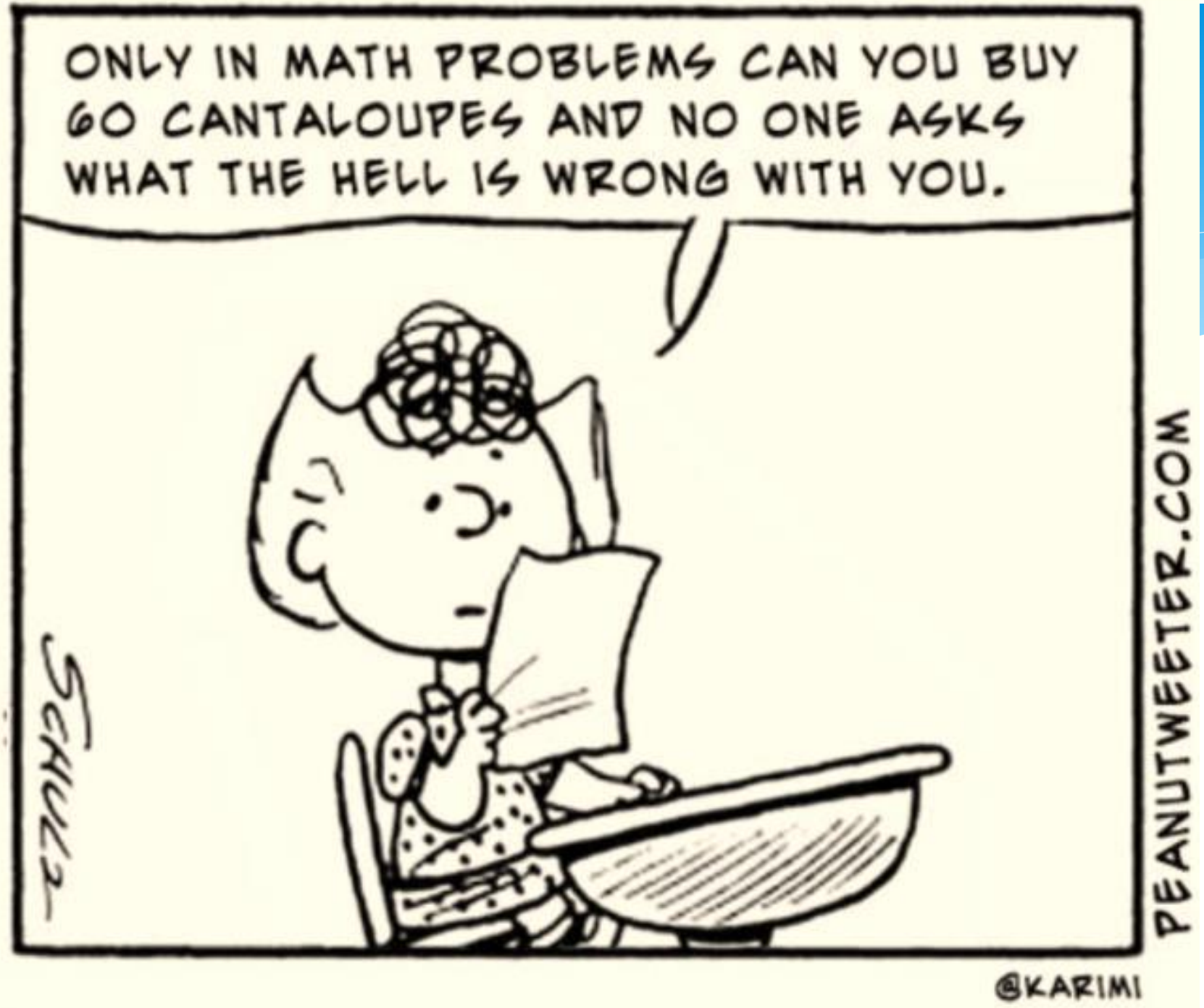
* Draw discourse arrows on your seating chart
  * Are all questions directed to you?
* Do you repeat student questions?
* Do you present fully worked problems?
* Do you follow dead end suggestions?

The Teacher's Role

* "Shifts from conveyer of information to one of moderator and observer of students' thinking" (Maher & Martino, 1996, p. 197).
* Connects students to the mathematics and methods they need to solve problems



Magic Gopher

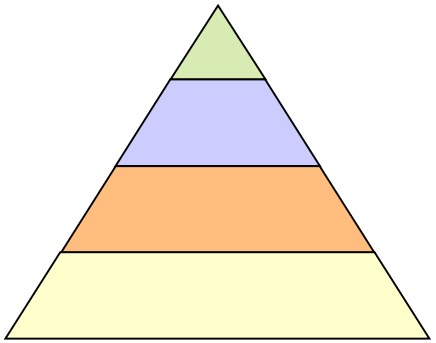

ONLY IN MATH PROBLEMS CAN YOU BUY 60 CANTALOUPES AND NO ONE ASKS WHAT THE HELL IS WRONG WITH YOU.
SCHULZ
PEANUTWEETER.COM
@KARIMI

Ideas


WolframAlpha computational knowledge engine
Enter what you want to calculate or know about:
Green Lake, WI and Middleton, WI
Examples
Random

Four Levels of Cognitive Demand

* Memorization
* Procedures Without Connections
* Procedures With Connections
* Doing Mathematics



* Stein, Smith, Henningsen, and Silver 2000, p. 16

Four Levels of Cognitive Demand
Memorization

* Reproducing a fact learned from memory
* Not ambiguous
* No connection to concepts or underlying meaning

2 + 7 =

Represent ¼ as a decimal and a fraction

Four Levels of Cognitive Demand
Procedures without Connections

* A recipe to follow
* Low cognitive demand
* Little ambiguity
* Focused on the answer, not the process

Davis wants to tile his rectangular room which is 8 feet by 10 feet. How many square feet of tile will he need to purchase?

Four Levels of Cognitive Demand

Procedures WITH Connections

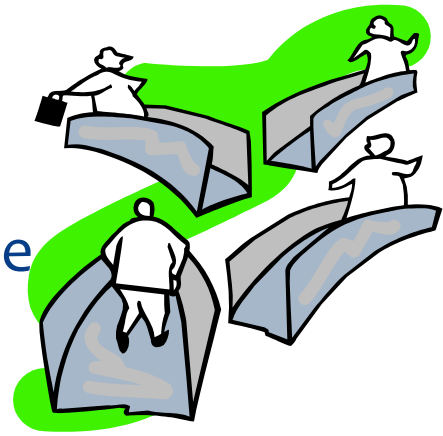
* Focus on the procedure to create deeper understanding
* A path to follow may be suggested
* Often multiple representations are used
* Requires mental effort

Using base-ten blocks, show that .17 is less than .2

Jon lists his house with a realtor that takes a 4% commission. He owes $92,000 on his house. How much does Jon's house need to sell for to cover his debt?

Four Levels of Cognitive Demand
Doing Math

* Complex thinking
* No path or recipe to follow
* Students need to access necessary knowledge
* Cognitively demanding



In my bag of candy ½ of the pieces are red, 1/3 are blue, the remaining 21 are white. How many pieces are in the bag?

Four Levels of Cognitive Demand

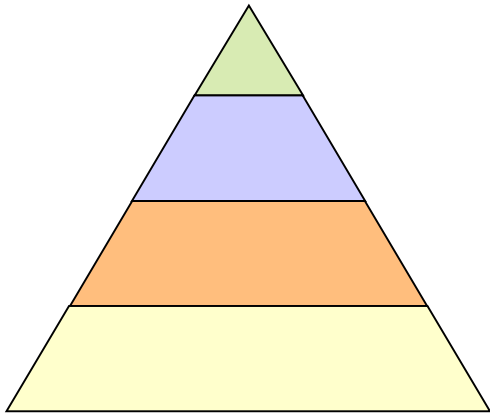
Memorization

Procedures Without Connections

Procedures With Connections

Doing Mathematics

* How would you categorize what we've done?
  * 4 Odd Numbers
  * Pattern blocks
  * Ryan's Card Piles
  * Magic Gopher
  * Wolfram Alpha



Four Levels of Cognitive Demand

Memorization

Procedures Without Connections

Procedures With Connections

Doing Mathematics

* Students who perform best on project-based problem solving were in classrooms in which tasks were implemented at high levels of cognitive demand
  Leutzinger, L. ICTM Journal, 2010
* Good teaching demands a blend of all four levels

Ideas from Dan Meyer

https://www.youtube.com/watch?v=BlvKWEvKSi8

* How much water do you use in the shower?
  * How long does it takes to fill up a gallon
  * Average amount of time spent in the shower
* Exploring surface area using cheese blocks of same volume but different surface areas
  * Which takes longest to melt in the microwave
* Using plastic cups… how many cups will you have to stack to be the height of your teacher

Ideas

* What could you do with newspapers
  * Work with place value
  * Purchase car
* Give students a large amount of frequent flier miles to plan a trip around Europe with.
  * What topics could you cover with this?
* What can you do with census data?

Kristin Kanaskie 2011 Fall ICTM Journal p. 4

Ideas

* Visit a creek
  * Estimate depth
  * Estimate water flow (Math's Teacher, May 2012)
* A bag of oranges
  * Volume
  * Peel it to find surface area (Math's Teacher, Oct. 2011)
  * Stack oranges for space

Ideas

* Turning everyday living activities into math
* Have students record, survey, analyze, and collect data:
  * number of texts send/received a day
  * hours of sleep
  * hours of T.V.
  * hours of favorite activity...
* Students can collect data in a journal and expand using graphs and calculations

Ideas

* Design a mini-golf course
* Pull up Hy-Vee's on-line add
* Have students present a problem of the week
  * Google doc of problems

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